Deformed Energy Dispersion In Neutrino Oscillation

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Abstract

Recently there has been an increasing number of anomalies in high energy neutrino oscillations. To explain this there has been proposed modifications to the energy dispersion relation to explain the anomalies. We will be looking at generic class of modified energy dispersion relation and explore how it affects the probability of 2-flavour neutrino oscillations and the possibilities that stem from the modification, thus offering an explanation for the anomalies.

1. Introduction

In 1968 Ray Davis conducted an experiment to detect neutrinos from the sun with a tank of chlorine in Homestake mine in south Dakota, USA. This is achieved by counting the number of argon atoms produced from the reaction of chlorine with neutrinos [1]. However the number of argon atoms collected was only a third of what was predicted at that time by John Bahcall [2] and hence resulting in the solar neutrino problem. This was solved also in 1968 by Bruno Pontecorvo as he suggested that neutrinos oscillated from 1 flavour to another while traveling, thus explaining the anomaly [3].

Then in 2015, the Nobel Prize for Physics was awarded to Takaaki Kajita in Japan and Arthur B. McDonald in Canada for their discoveries in 2 flavour neutrino oscillations further strengthening the theory that neutrinos have masses. However, current models are not able to explain anomalies in experimental data concerning neutrinos at high energy thus, a need for a modified to the dispersion relation is required to explain the anomalies detected in the LSND and MinibooNE experiments [4].

In this paper I will explore one of the proposed modified 2 flavour neutrino oscillation dispersion relation

$$E^2 = p^2c^2 + m^2c^4 + \eta p^2 E^\alpha; |\eta| \ll 1; \alpha \in \Re$$

from Ref. [5] with η as the modifying term and explore the differences between the current model and proposed modifications, using data from the MINOS collaboration, Super-Kamiokande and KamLand experiments. Here η is dimensionful parameter with $[\eta] = \left[\frac{1}{p^2 E^{\alpha-2}}\right]$ depending on α

2. Method

2.1 Neutrino Oscillation in Conventional Energy Dispersion Relation (CEDR)

The Standard neutrino oscillation starts with the following equations [6]

$$P_{\nu_1->\nu_2} = \sin^2(2\theta)\sin^2(\frac{E_1 - E_2}{2\hbar}t) \quad \text{(Eqn 2.1)}$$

where the probability of an electron neutrino oscillating into a muon neutrino is given by the product of the mixing angle sin(2θ) and the sine of the energy difference multiplied by the time elapsed from its creation divided by the reduced plank constant squared.

To find the energy difference we use the equation

$$E^2 = p^2c^2 + m^2c^4 \quad \text{(Eqn 2.2)}$$

By expanding Eqn 2.2 to the leading order in $0\left(\frac{m^2c^2}{p^2}\right)$

$$E = cp\sqrt{1 + \frac{m^2c^2}{p^2}}$$

$$E = cp\left(1 + \frac{m^2c^2}{2p^2} + \dots\right)$$

$$E \approx cp + \frac{m^2c^3}{2p} + 0\left(\frac{m^2c^2}{p^2}\right)$$

$$E = cP + \frac{m^2c^3}{2P} \quad \text{(Eqn 2.3)}$$

As the neutrino mass is insignificant to its kinetic energy we can assume that $E \approx cP_e \approx cP_\mu$ where P is kept constant. Thus, Eqn 2.3 can be rewritten as

$$E \approx cP + \frac{m^2c^4}{2E}. \qquad \text{(Eqn 2.4)}$$

By taking the difference $E_1 - E_2$ in energy levels of the electron neutrino ($\nu_1 = \nu_e$) and the muon neutrino ($\nu_2 = \nu_\mu$) using Eqn 2.4 we obtain

$$E_1 - E_2 = \frac{\Delta m_{12} c^4}{2E}. \qquad \text{(Eqn 2.5)}$$

Inserting Eqn 2.5 into Eqn 2.1 we obtain

$$P_{\nu_1 -> \nu_2} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2 c^4}{4E\hbar} t\right).$$

For this paper we will rewrite the equation with a distance variable for inputing experimental values where the distance a neutrino travel $z \approx ct$, so we get

$$P_{\nu_1 -> \nu_2} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2 c^3}{4E\hbar} z\right). \qquad \text{(Eqn 2.6)}$$

2.2 Neutrino Oscillation in Modified Energy Dispersion Relation (MEDR)

With the CEDR, we are unable to explain some anomalies found in high energy neutrino experimental data thus researchers have theorised some modifications to the dispersion relation. We will be focusing on one of such modifications from Ref. [3] in the form of

$$E^2 = p^2c^2 + m^2c^4 + \eta p^2 E^\alpha; |\eta| \ll 1; \alpha \in \Re$$

For simplicity we consider a specific example in which $\alpha = 2$ and $[\eta] = \left[\frac{1}{p^2}\right]$. Following the same steps as section 2.1 we obtain

$$E_1 - E_2 = \frac{\Delta m_{12} c^4}{2E} + \frac{\Delta \eta_{12} E^3}{2c^2} \qquad \text{(Eqn 2.7)}$$

Substituting Eqn 2.7 into Eqn 2.1 we get

$$P_{\nu_1 -> \nu_2} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2 c^3}{4E\hbar} z + \frac{\Delta \eta_{12} E^3}{4\hbar c^3} z\right) \qquad \text{(Eqn 2.8)}$$

Or equivalently we can express the oscillating behaviour in Eqn 2.8 as

$$P_{\nu_1 -> \nu_1} = 1 - \sin^2(2\theta)\sin^2\left(1.905 \frac{\Delta m_{12}^2[\mathrm{eV}^2] c^3}{4E[\mathrm{GeV}]\hbar} z[\mathrm{Km}] + \frac{\Delta \eta_{12} E^3[\mathrm{GeV}^3]}{4\hbar c^3} z[\mathrm{Km}]\right) \qquad \text{(Eqn 2.9)}$$

In Eqn 2.9 the oscillation phase is now dependent on the the deformed parameter $\Delta\eta_{12}$ the energy and the length of propagation. To recover the conventional oscillation phase we can take the limit when $\Delta\eta_{12} = 0$. Additionally, with the term $\left(\frac{\Delta\eta_{12}E^3}{4\hbar c^3} z\right)$ it allows the possibility of $\Delta m_{12}^2 = 0$ and hence allow neutrino flavours to have degenerate mass or $m_1 = m_2 = 0$ as a special case

3. Results

Using data from Ref [7] as presented in table 1, we will be plotting graphs to see the effects of the modification and to calculate the upper bound for $\Delta\eta$ by equating

$$\frac{\Delta m_{12}^2 c^3}{4E\hbar} z = \frac{\Delta\eta_{12} E^3}{4\hbar c^3} z.$$

Thus the bound for $\Delta\eta$ for different types of neutrino will be

$$\Delta\eta = \frac{\Delta m^2 c^6}{E^4}$$

$$\approx 3.5 \times 10^{30} \frac{\Delta m^2 [\mathrm{eV}^2]}{E^4 [\mathrm{MeV}^4]} \qquad \text{(Eqn 2.9)}$$

Data values

Type of neutrino	Δm^2	$\sin^2(2\theta)$	z
Solar (1,2)	$7.59 \times 10^{-5} \mathrm{eV}^2$	0.859	1,500,000 Km
Atmospheric (2,3)	$2.45 \times 10^{-3} \mathrm{eV}^2$	1.00	5000 Km
Beamline (1,2)	$2.45 \times 10^{-3} \mathrm{eV}^2$	0.0746↔0.0159	735 Km

Table.1 oscillation parameter used for the following graphs in sections 3.1, 3.2, 3.3 and 3.4

3.1 Solar Neutrino Oscillation ($\nu_e \to \nu_\mu$)

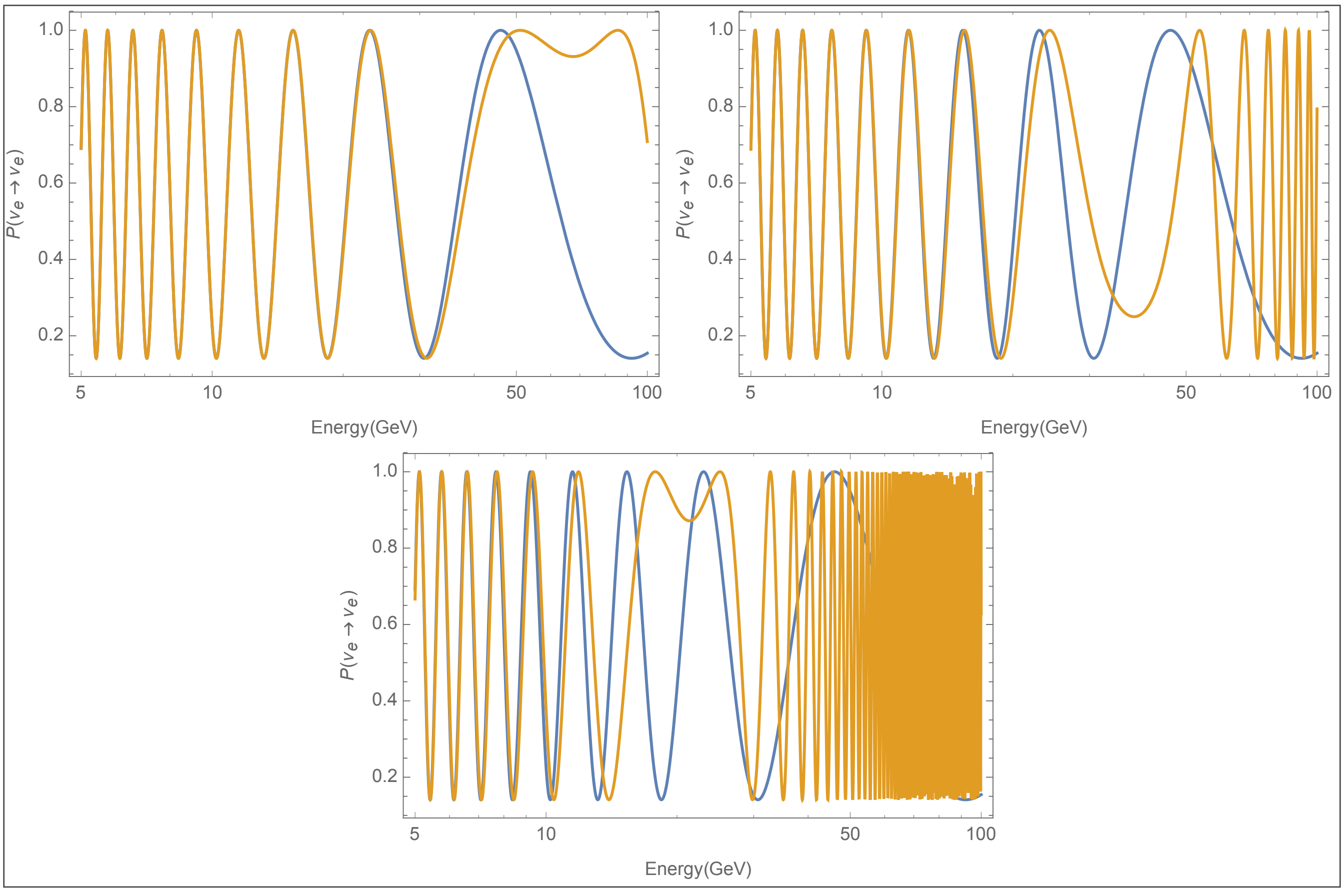

Fig 3.1 The graphs with magnitude of Δη set to 10^{-30} (top left), 10^{-31} (top right), 10^{-32} (Bottom). The blue line shows the standard oscillation phase while the yellow shows the modified oscillation phase.

Using Eqn 3.1 we can find the upper bound for term modification term η with the lowest solar neutrino energies of around 0.4MeV

$$\eta_{max}^{solar} \leq 1.04 \times 10^{28} (\mathrm{kg^{-2} m^{-2} s^{2}})$$

where η_{max}^{solar} is the maximum value η can take in solar neutrino instances.

3.2 Atmospheric Neutrino Oscillation ($\nu_\mu \rightarrow \nu_e$)

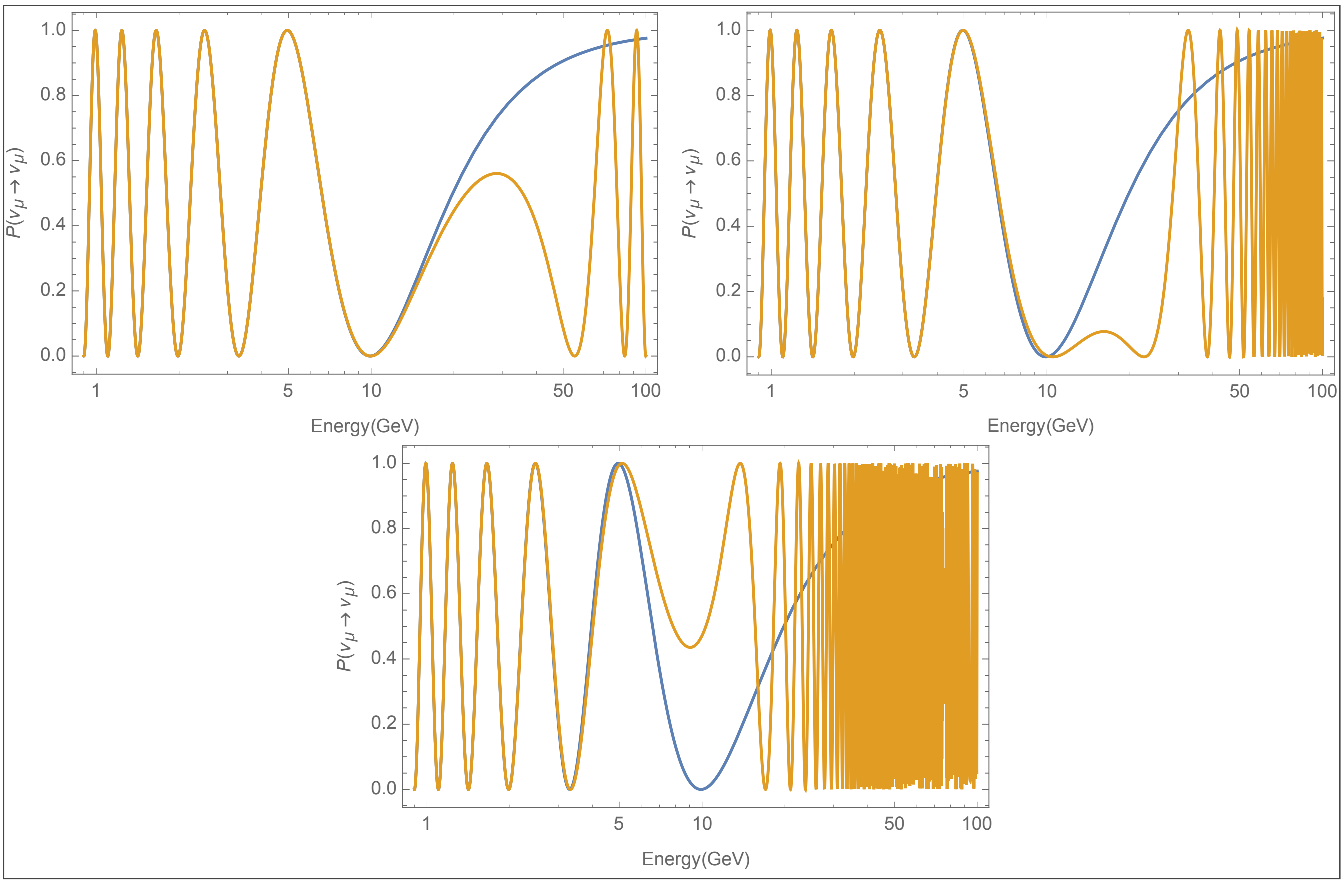

Fig 3.2 The graphs with magnitude of Δη set to 10^{-27} (top left), 10^{-28} (top right), 10^{-29} (Bottom).

Atmospheric neutrinos have energy levels around 1GeV so the upper round for η would be

$$\eta_{max}^{atm} \leq 8.58 \times 10^{15}(\mathrm{kg}^{-2}\mathrm{m}^{-2}\mathrm{s}^{2})$$

Where η_{max}^{atm} is the maximum value η can take for atmospheric neutrinos

3.3 Beamline Neutrino Oscillation ($\nu_\mu \rightarrow \nu_e$)

As the uncertainties for $\sin^2 2\theta$ value for beamline neutrinos are very high compared to its value there will be a need to consider the range of values in which $\sin^2 2\theta$ can take resulting in it having 2 values, showing the upper and lower limit.

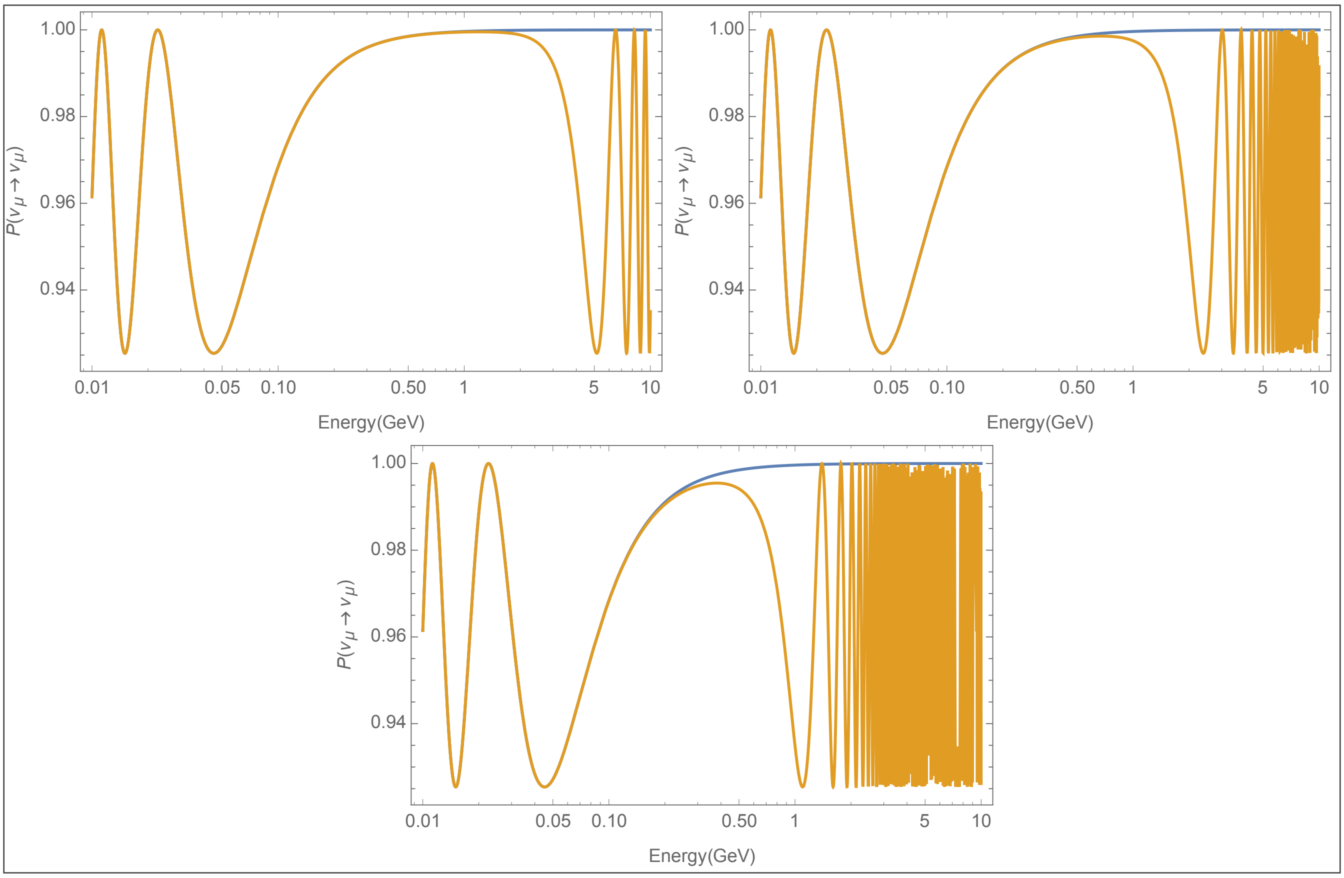

Fig 3.3 The graphs of $\mathrm{Sin}^2 2\theta=0.0746$ with magnitude of $\Delta\eta$ set to 10^{-23} (top left), 10^{-24} (top right), 10^{-25} (Bottom).

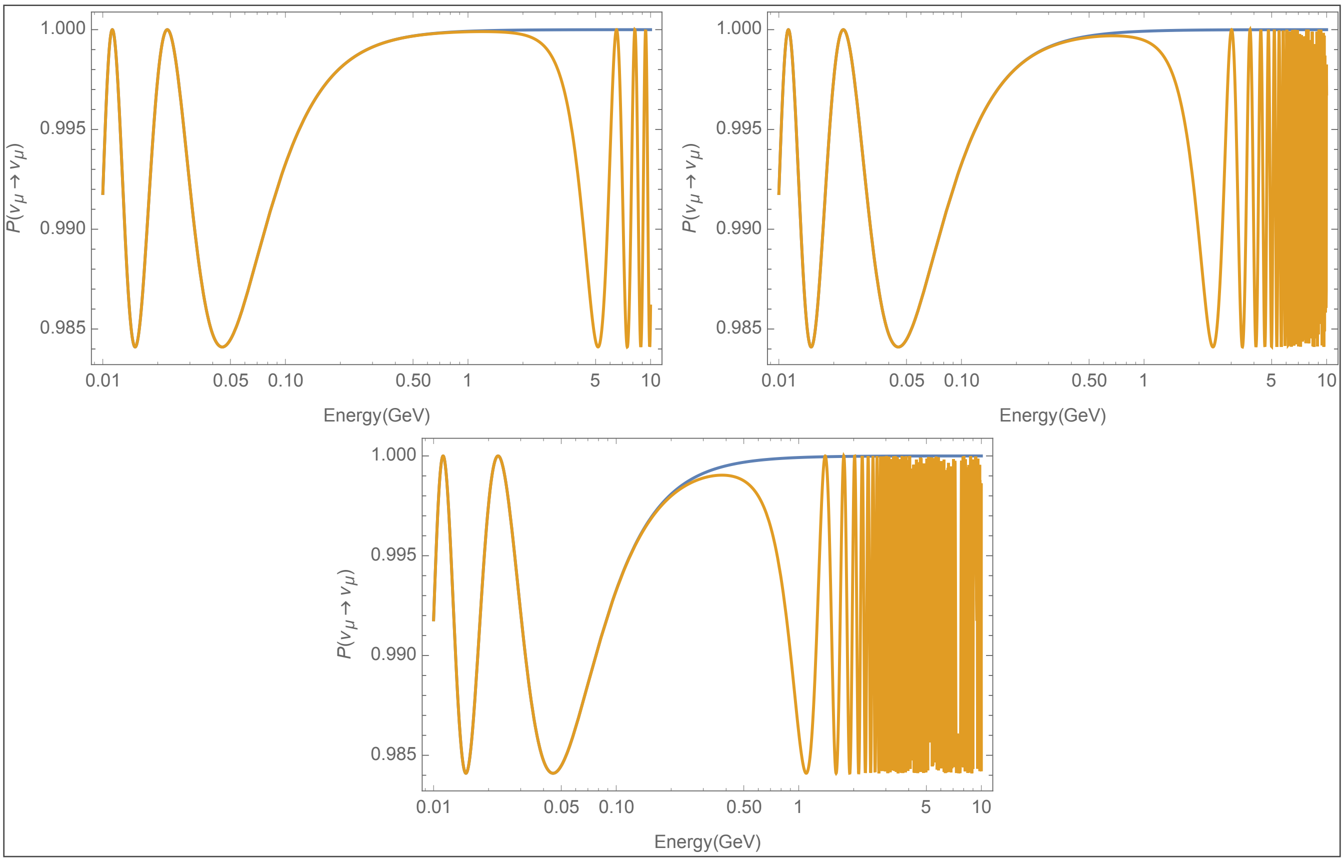

Fig 3.4 The graphs of $\mathrm{Sin}^2 2\theta$=0.0159 with magnitude of $\Delta\eta$ set to 10^{-23} (top left), 10^{-24}(top right), 10^{-25} (Bottom).

The Neutrinos at the Main Injector (NuMI) is the source of the MINOS detector producing neutrinos with lowest energies of around 2 GeV thus the upper bound for η would be

$$\eta_{max}^{beam} \leq 5.36 \times 10^{14} (\mathrm{kg}^{-2}\mathrm{m}^{-2}\mathrm{s}^{2})$$

Where η_{max}^{beam} is the maximum value η can take for beam neutrinos

3.4 possibility of $\Delta m^2 = 0$

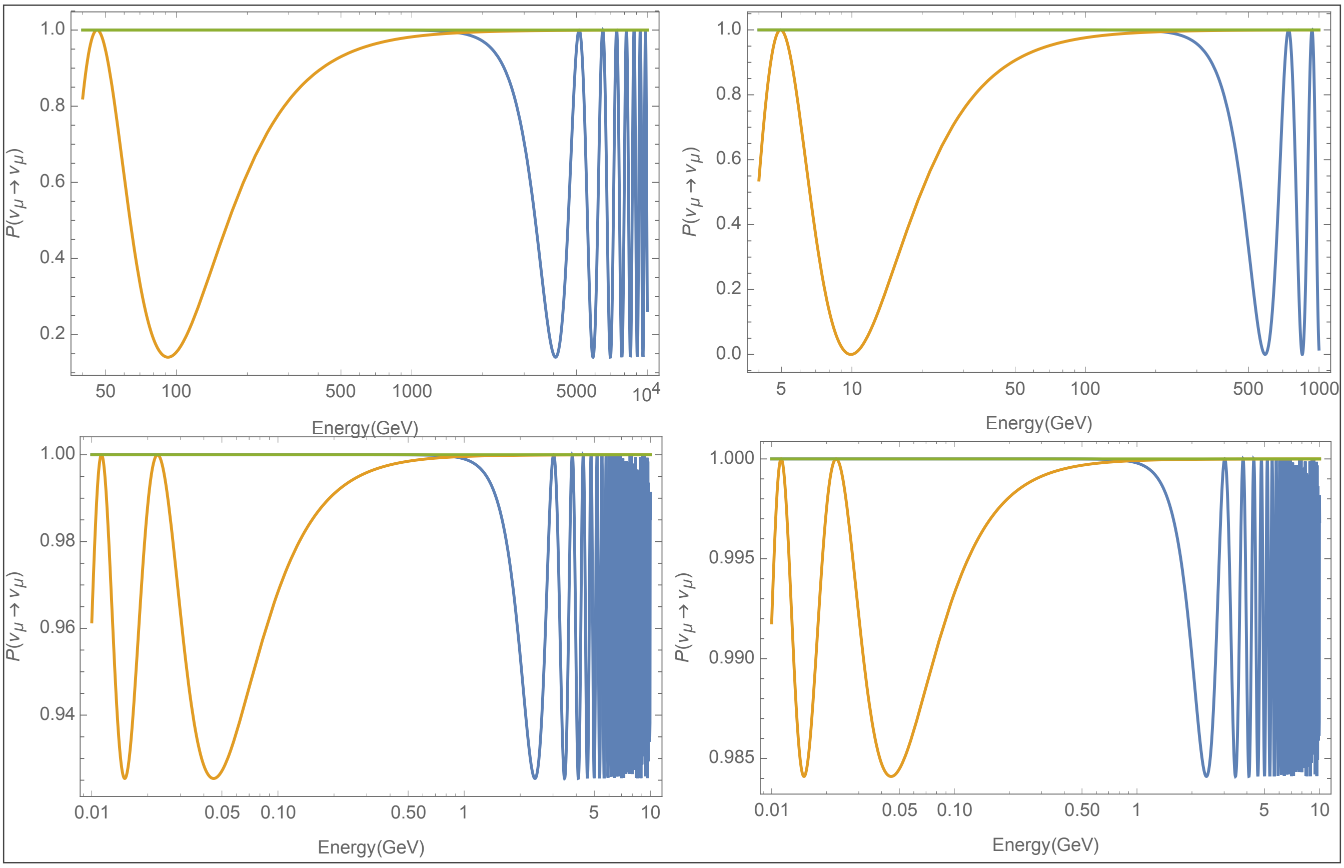

Fig 3.5 The graphs of MEDR of neutrinos. Green line represents MEDR Δm^2=0 and magnitude ofΔ η=0, trivially meaning that there is no oscillation. Orange line represents the MEDR with magnitude of $\Delta\eta$=0. Blue line represents MEDR with Δm^2=0 with different magnitudes of $\Delta\eta$. Solar neutrinos (top left) magnitude of $\Delta\eta$=-37, Atmospheric neutrino (top right and left) magnitude of $\Delta\eta$=-32, Beam neutrino (bottom right and left) magnitude of $\Delta\eta$=-24. Bottom left is the higher limit of $\mathrm{Sin}^2 2\theta$ for beam neutrinos and bottom right for lower limit

3.5 Analysis of results

From section 3 we can see that the MEDR affects neutrino oscillations only at high energies of their type while not affecting the oscillating phase at low energy levels depending on the magnitude of $\Delta\eta$. From another perspective, MEDR may also allow neutrino oscillation without mass difference which is essential in the CEDR. From section 3.4 we can see that with the CEDR at high energies with respect to the baseline, mass difference and mixing angle, the neutrino will stop oscillating thus the MEDR can offer a solution to explain neutrino oscillations at high energies. With this Neutrino Oscillations can be served as a useful probe to tell the physical range of values of η is allowed in theory. This very essential to the validity of the quantum nature of space time Eqn (1.1) is motivation from quantum gravity theories.

4. Conclusion

4.1 Results

In this report the objective is to test whether a MDR would offer explanation to anomalies in high energy neutrinos. To achieve this we first derived the modified oscillation phase from the MDR, and in our case we formed 5 variables, Δm^2, $sin^2 2\theta$, $\Delta\eta$, E and z, with this we then explored how the MDR would affect the the oscillating phase of different types of neutrinos, which we found out that at low energies the oscillation phase of the MDR is same as the CDR and the effects will only show at high energy levels. We also explored that at high energies where the CDR suggests that the neutrinos no longer oscillate The MDR would still be oscillating with increasing frequency which is highly unlikely.

4.2 Future Work

This paper explores only one of such MDRs, there are numerous of other proposed MDRs thus there is a need for more precise data to make sense of the different MDRs proposed and to enter modify the dispersion relations, offering more precise predictions.

References

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